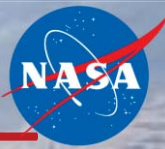




Fractional Order Modeling of Atmospheric Turbulence – A More Accurate Modeling Methodology for Aero Vehicles

Abstract:

This presentation has been prepared for the Aerospace Controls and Guidance Systems Committee meeting, which takes place in Cleveland Ohio on Oct. 14-17, 2014. The presentation covers a recently developed methodology to model atmospheric turbulence as disturbances for aero vehicle gust loads and for controls development like flutter and inlet shock position. The approach models atmospheric turbulence in their natural fractional order form, which provides for more accuracy compared to traditional methods like the Dryden model, especially for high speed vehicle. The presentation provides a historical background on atmospheric turbulence modeling and the approaches utilized for air vehicles. This is followed by the motivation and the methodology utilized to develop the atmospheric turbulence fractional order modeling approach. Some examples covering the application of this method are also provided, followed by concluding remarks.



Fractional Order Modeling of Atmospheric Turbulence – A More Accurate Modeling Methodology for Aero Vehicles

George Kopasakis

**NASA Glenn Research Center
Cleveland Ohio**

**Aerospace Control and Guidance Systems Committee
Oct. 15-17 Cleveland, Ohio**



Outline

- **Background**
- **Fractional Order Modeling Motivation**
- **Fractional Order Modeling Approach**
- **Applications**
- **Conclusions**



Background

- Traditionally atmospheric disturbance applied to air vehicles structural component designs to handle loads induced by severe gusts.
- Collection of atmospheric turbulence data:
 - Gust **accelerometer** data collected by NASA & Air Force (1956-75) to establish a statistical data base (Ehernberger and Love, 1975)
 - Gust velocity & directionality measurements by NASA in late 80's
 - Gradients of temperature transients over 4 years in early 90's by Concord.
- For propulsion system the concern is freestream flow velocity, temperature, and pressure disturbances entering the inlet and angularity of the flow (coupled w/ additional pitch, yaw and roll disturbances), which contribute to inlet unstart.
- Today for the High Speed Project (HSP) the concern is gust loads and AeroPropulsoServoElasticity (APSE) – coupling of propulsion and structural dynamics.

Background

- Historically conditional probability analysis is applied; that event y will take place provided that a certain magnitude atmospheric turbulence E is encountered (based on atmospheric data)

$$P(y|E) = P(E) \times P(y|E)$$

- Exceedance rates are typically developed for aircraft designs based on probabilistic models (specific to a particular design)

$$R(V) = P_t N_{zero} \int_0^{\infty} e^{-\frac{V^2}{2r\sigma^2}} P(\sigma) d\sigma$$

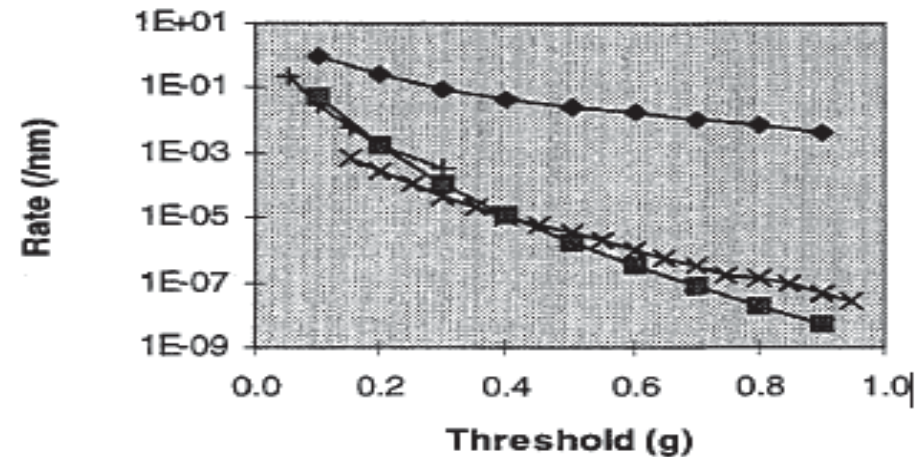
$$r = \frac{\int_0^{\infty} T^2(k) S(k) dk}{\int_0^{\infty} S(k) dk}$$

$$N_{zero} = \sqrt{\frac{\int_0^{k_{cutoff}} k^2 T^2(k) S(k) dk}{\int_0^{k_{cutoff}} T^2(k) S(k) dk}} \frac{1}{\pi}$$

$$P(\sigma) = \frac{1}{\ln(10) \sigma \sqrt{2\pi}} e^{-\frac{(2 \log(\frac{\sigma}{\sigma_0}))^2}{2s^2}}$$

$$\sigma^2 = 1.35 \epsilon^{\frac{2}{3}} L^{\frac{2}{3}}$$

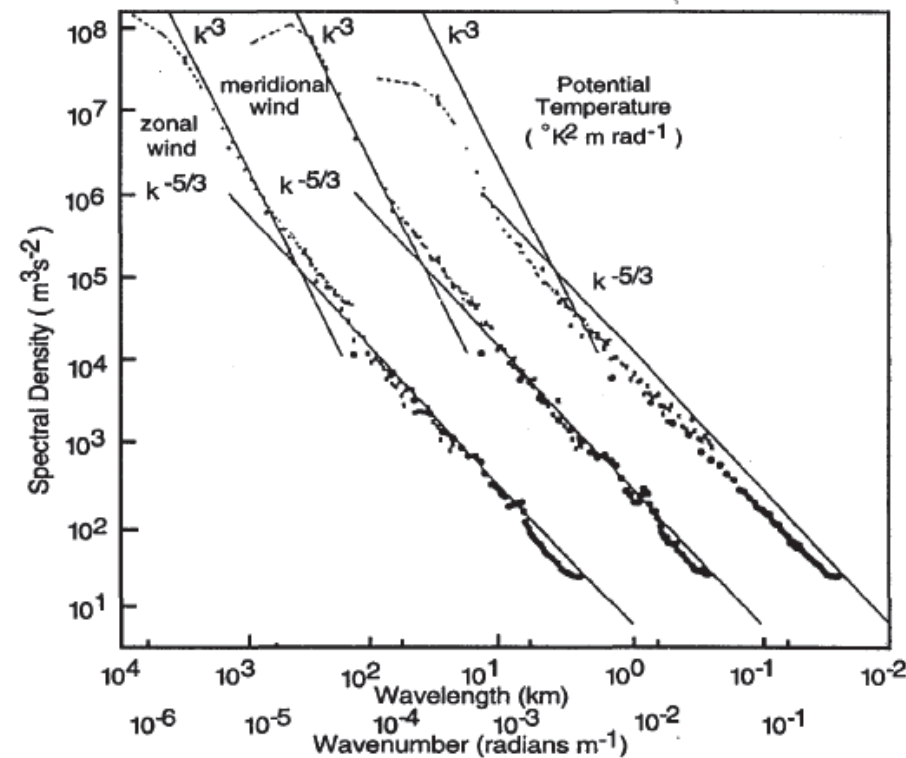
747 Exceedance comparison



- x— 747 data NLR-TP-93535-L
- +— 747 data CR-181909
- epsilon= 8.6E-5 width=.43
- ◆— NASP model @ 10 km

Background

- Atmospheric Turbulence models are based on the Kolmogorov spectrum - spectrum originally developed by Tatarski (1961). This spectrum has an energy that approaches infinity at low frequencies - Kolmogorov; Russian scientist first analyzed atm. turb. in 1940's .



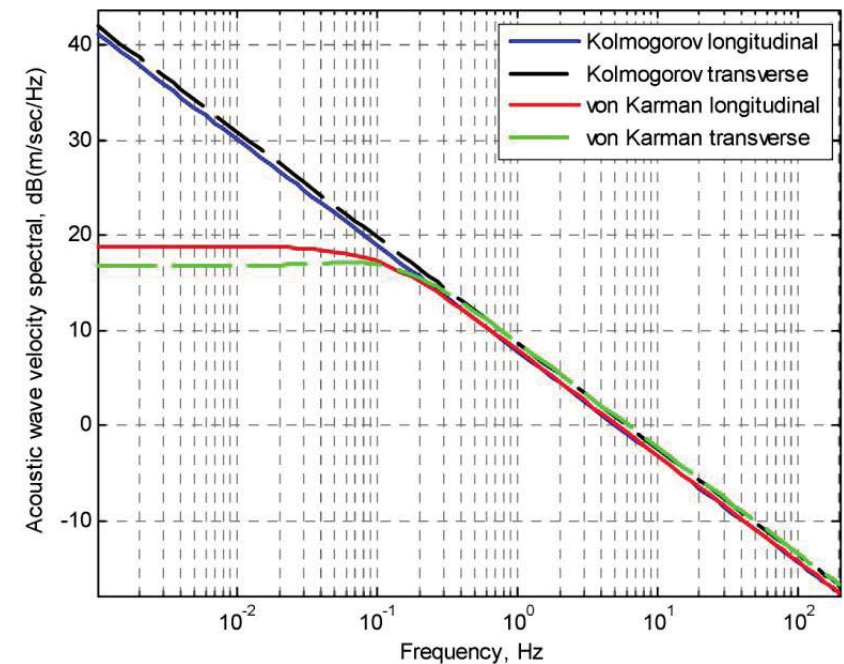
Wind and Potential Temperature Spectra as reported by Nastrom (1985). For clarity, meridional wind and temperature spectra have been shifted one and two decades to the right, respectively.

$$S_t(k) = \alpha_t \varepsilon^{2/3} k^{-5/3}$$

⇒ Kolmogorov Spectrum 1961 ⇒

$$S_{v,K}(k) = 2.7 \varepsilon^{2/3} L^{5/3} \frac{1 + \frac{8}{3} (1.339(2\pi)Lk)^2}{[1 + (1.339(2\pi)Lk)^2]^{11/6}}$$

von Karman spectrum, 1996 by Soreide & Tank



Acoustic wave velocity spectral comparisons for the Kolmogorov and von Karman spectral



Background

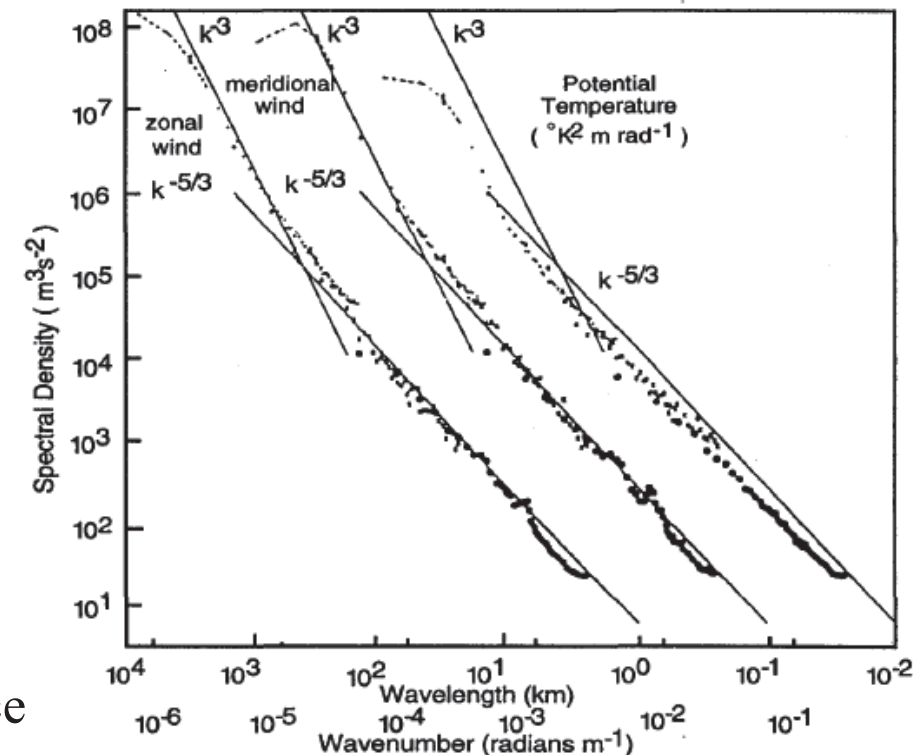
- The slope of the atmospheric turbulence spectrum changes at about 400 km turbulence length.
- Spectrum density of turbulence increases with decreasing wave number (increasing length of turbulence).
- Spectral also increases with increased turbulence levels – increased eddy dissipation rate, ε , (energy/mass/time).

$$\varepsilon = \nu * v_l'^2 / l^2$$

ν – kinematic viscosity
 v_l' – velocity fluctuation in atmospheric region of size l

$$S_t(k) = \alpha_t \varepsilon^{2/3} k^{-5/3}$$

- ε specifies the rate at which energy in the turbulence breaks down when a critical Reynolds number is exceeded until it completely dissipates, in the process leading to increase wind gusts.
- The assumption is that in worst case turbulence a critical Reynolds no. is exceeded $Re = \frac{\rho v^2 L^2}{\mu \nu L}$, $L = 0.07$ size of turbulence



Background



- The Soreide and Tank von Karman spectrum can be used as the amplitude of sinusoids at different frequencies to develop a time domain spectrum of certain atmospheric turbulence intensity of eddy dissipation rate, ε , and integral length scale, L .

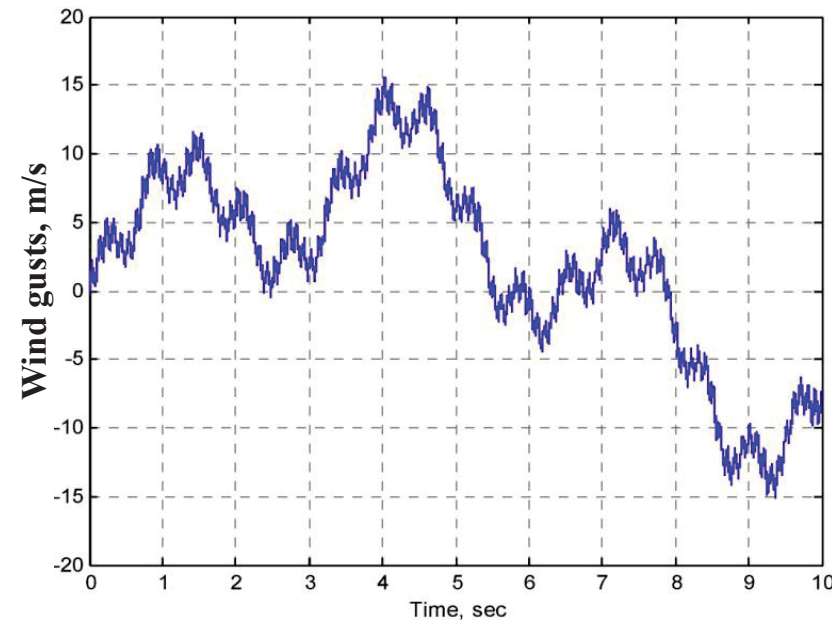
$$S_{L, VK}(k) = 2.7 \varepsilon^{2/3} L^{5/3} \frac{2}{[1 + (1.339(2\pi)Lk)^2]^{5/6}} = A_i(\varepsilon, L, f_i) \Rightarrow \sum_{i=1}^n A_i(\varepsilon, L, f_i) \sin(2\pi f_i t)$$

$$f_{range} = \frac{V_{vehicle}}{\lambda_s}, f_n \leq f_{range}, \lambda_s \sim 25\text{m} - \text{wavelength of smallest turbulence structures w/ increased gusts}$$

- Alternatively, the Dryden model was developed to simulate atmospheric turbulence (Hoblit, 1988)

$$\text{ex. } \Phi_{u_g}(\Omega) = \sigma_u^2 \frac{2L_u}{\pi} \frac{1}{1 + (L_u\Omega)^2} \Rightarrow \frac{A}{\left(\frac{s}{\omega}\right)^2 + 1}$$

- The Dryden model is 2nd order instead of 5/3, which underestimates turbulence by 7 dB per frequency decade. Exact fractional TF not possible mathematically.
- Underestimation acceptable for lower speed vehicles, but can be problematic for higher vehicle speeds.

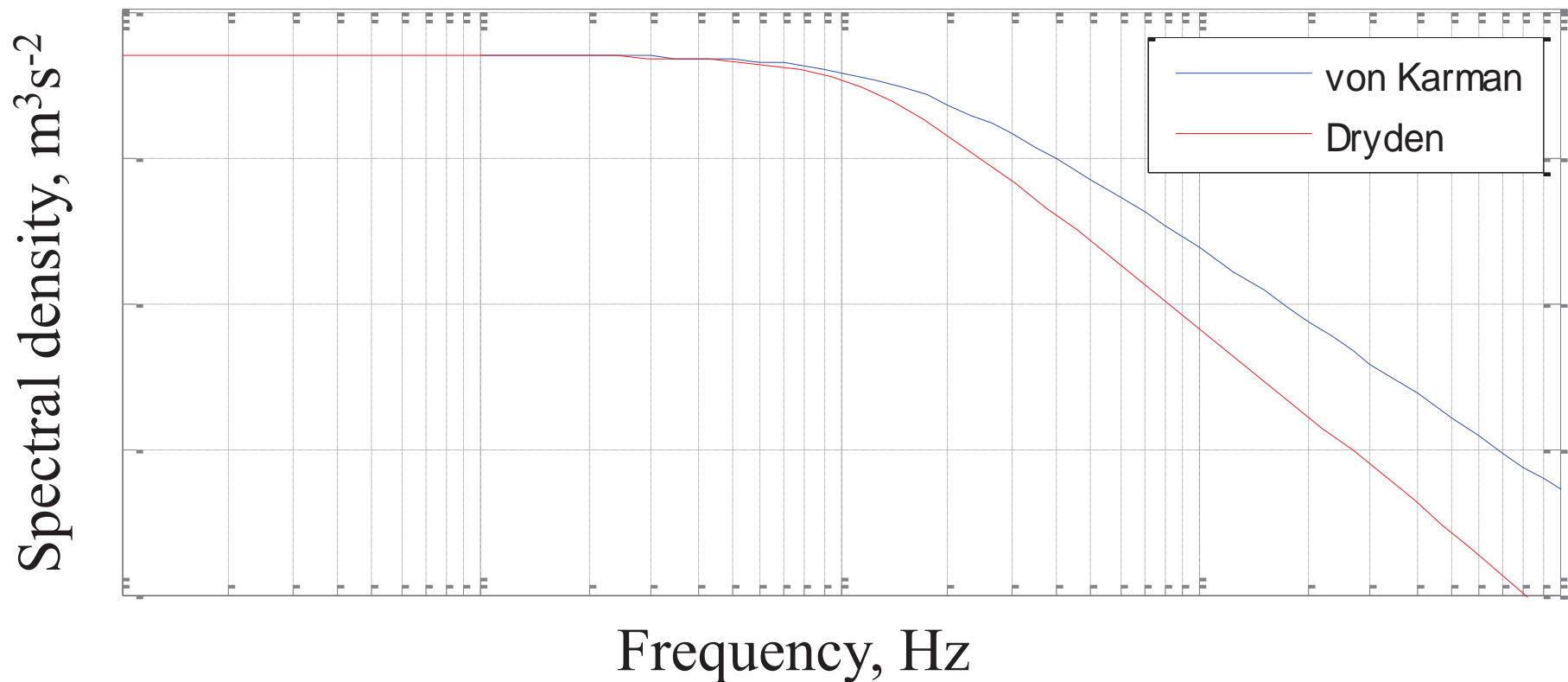




Background

Comparison between von Karman model (exact) and Dryden model (approximate)

- Underestimation of spectral components, 7 dB/dec deviation after 3db natural frequency.



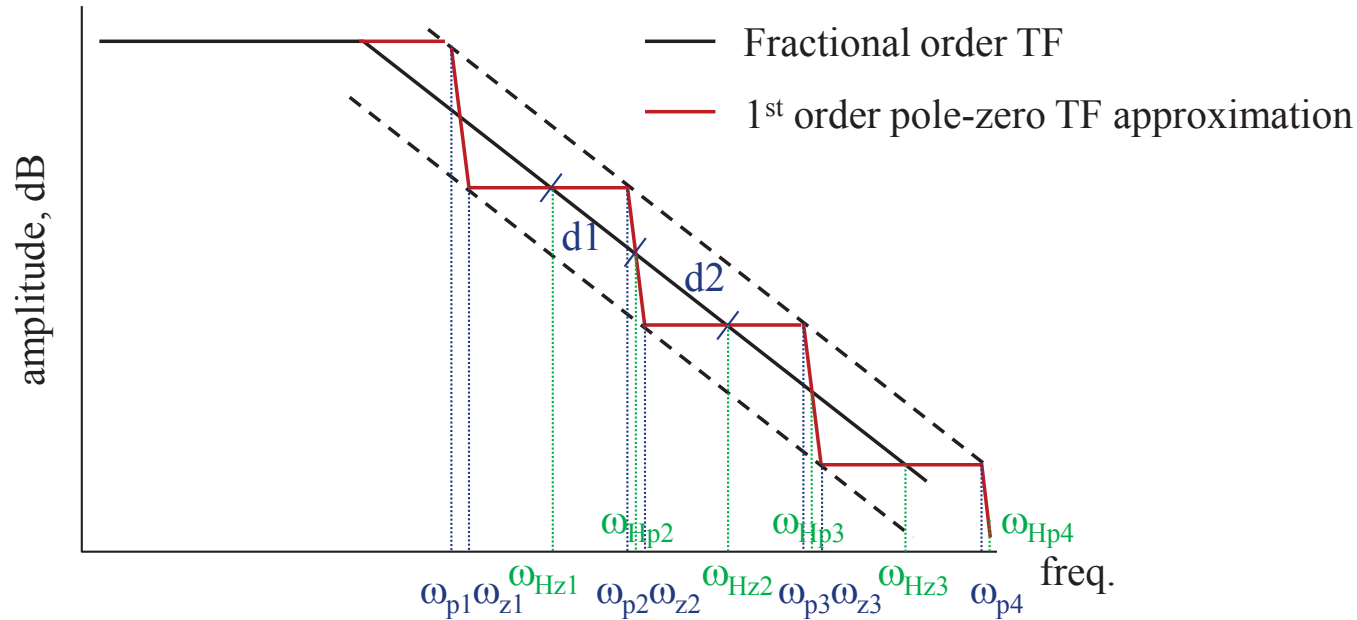


Fractional Order Modeling Motivation

- Because the Soreide – Tank model is more difficult to use and because the Dryden model significantly underestimates atmospheric turbulence for supersonic vehicles the need was identified to develop an accurate fractional order turbulence model in frequency domain.
 - This model acts as a low pass filter and only unity amplitude sinusoids are needed as inputs to simulate the atmospheric turbulence spectrum associated with turbulence.
- Also, instead of probabilistic models like exceedance rates, the motivation is to simulate certain worst case atmospheric turbulence and understand how the vehicle model or the controls will perform.
 - The reason is based on the assumption that if a vehicle encounters certain atmospheric turbulence conditions, breakdown due to an eddy dissipation rate associated with that turbulence (that generates peak gusts) disturbances will be a certainty.
 - The new model can still be used in conjunction with probabilistic calculations like exceedance rates.

Fractional Order Modeling Approach

- Estimate a fractional order TF utilizing 1st order pole – zero pairs (for fractional order less than 1) – ex. TF for 1 pole-zero pair, $G = \frac{(s/\omega_z + 1)}{(s/\omega_p + 1)}$



- The idea is as the density of pole zero pairs increase the stair case estimate will approach a straight line.
- The challenge is to properly locate the frequencies of the poles and zeroes and the intersections of these lines so that the stair case estimate is symmetrically located on top of the fractional order slope
 (e.g., $d1=d2\dots dn$) $\Rightarrow \omega_{pi,zi} = f(\omega_{p1}\dots\omega_{pi-1}, \omega_{z1}\dots\omega_{zi-1}, \omega_{Hpi}, \omega_{Hzi})$

Fractional Order Modeling Approach

1. Change units of spectrum to more convenient units (i.e., from $(\text{m/sec})^2/(\text{cycle/m})$ to $(\text{m/sec})/\text{Hz}$ and relate the wavenumber to the speed of the vehicle.

Kolmogorov Spectrum

$$S_t(k) = \alpha_t \varepsilon^{2/3} k^{-5/3}$$

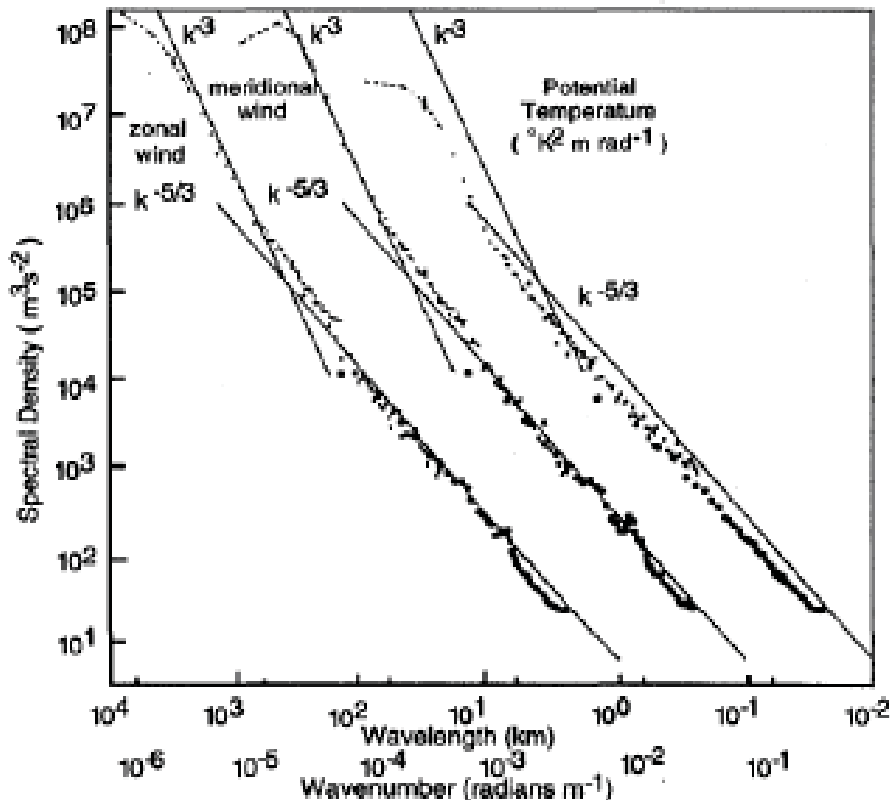


$$S_t(k) = \left(\alpha_t \varepsilon^{2/3} \left(\frac{f}{Ma} \right)^{-5/3} \right)^{1/3}$$

Von Karman Spectrum

$$S_{l,VK}(k) = 2.7 \varepsilon^{2/3} L^{5/3} \frac{2}{[1 + (1.339(2\pi)Lk)^2]^{5/6}} \Rightarrow S_{l,VK}(k) = \left(2.7 \varepsilon^{2/3} L^{5/3} \frac{2}{[1 + (1.339L \frac{2\pi f}{Ma_o})^2]^{5/6}} \right)^{1/3}$$

longitudinal

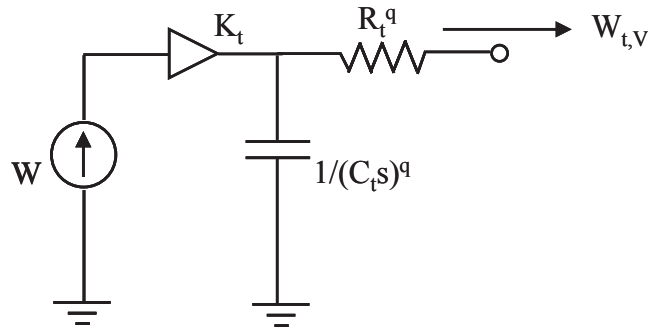


$$S_{v,VK}(k) = \left(2.7 \varepsilon^{2/3} L^{5/3} \frac{1 + \frac{8}{3}(1.339(2\pi)Lk)^2}{[1 + (1.339(2\pi)L \frac{2\pi f}{Ma_o})^2]^{11/6}} \right)^{1/3}$$

\Rightarrow
Transverse (vertical)

Fractional Order Modeling Approach

2. Develop a fractional order circuit model for the each type of disturbance, t , based on the Von Karman spectral and derive the circuit parameters of the model.



Circuit Equivalent of Atmospheric Disturbances

Resistance: $R_t = 1.339(2\pi)(a_t \varepsilon^{2/3})^{1/x} L$

Capacitance: $C_t = \frac{1}{(a_t \varepsilon^{2/3})^{1/x} (2\pi M a_o)}$

Natural Frequency: $\omega_n = \frac{K_{\omega n}}{R_t C_t}$

Gain: $K_t = f(\varepsilon, L)$

$x=5/3$, $r=1/3$ for wind gusts and $1/2$ for Temp. & Pres., $q=xr$

Type of disturbance t – wind gust longitudinal, wind gust transverse, temp., pres.

Fractional Order Modeling Approach

3. Verify that the von Karman type circuit model closely approximates the Kolmogorov model, apply scaling as necessary for proportional gain.

Example circuit
Approximation:

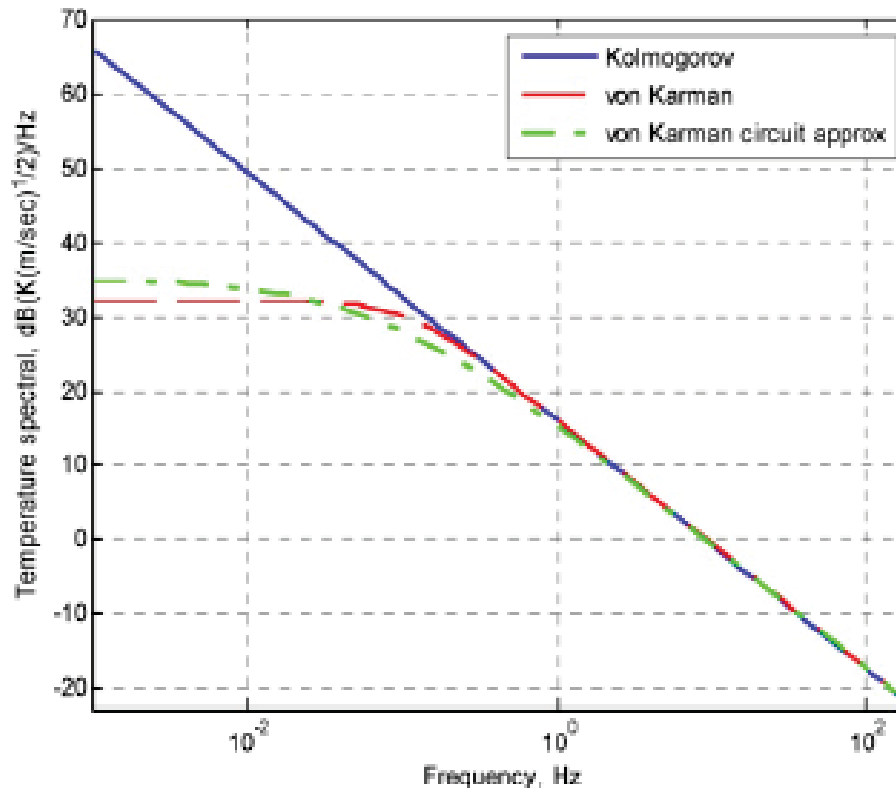


Figure B.7.—Kolmogorov and von Karman spectral for temperature and its von Karman circuit approximation ($\epsilon = 8.6\text{e-}5 \text{ m}^2/\text{sec}^3$, $L = 762 \text{ m}$).

- Circuit approximation works sufficiently well. However, its filter dynamics cannot be simulated because the circuit is fractional order.



Fractional Order Modeling Approach

4. Instead circuit model and its parameters are used as the basis to derive formulations for fractional order TF fit based on first order pole-zero pairs.

Derived TF of fractional order approximation:

(W_t are unit amplitude sinusoids distributed up to frequency, f_{range})

$$W_{t,o} \cong K_{t,fit} \frac{\prod_{i=1}^{m_z} (s / \omega_{zi} + 1)}{\prod_{i=1}^{m_p} (s / \omega_{pi} + 1)} W_t$$

Pole frequencies: $\omega_{pi} = \omega_f (10^{\eta q} - 1)^{1/q}$

$$\omega_{pi} = \frac{K_{\omega pi} \omega_{Hpi} \prod_{j=1}^{i-1} (\omega_{Hpi} / \omega_{pi-j} + 1)}{10^{\eta(2i-1)*q} \prod_{j=1}^{i-1} (\omega_{Hpi} / \omega_{zi-j} + 1) - 1}, \quad i=2, 3, \dots, m_p$$

Zero frequencies: $\omega_{zi} = \frac{K_{\omega zi} \omega_{Hzi} \prod_{j=1}^{i-1} (\omega_{Hzi} / \omega_{zi-j} + 1)}{10^{-2\eta i * q} \prod_{j=1}^i (\omega_{Hzi} / \omega_{pi} + 1) - 1}, \quad i=1, 2, \dots, m_z$

Intersection frequencies

that guarantee symmetry:

$$\omega_{Hpi} = \omega_f (10^{\eta q(2i-1)} - 1)^{1/q}$$

$$\omega_{Hzi} = \omega_f (10^{2\eta qi} - 1)^{1/q}$$

Calculation of other supporting parameters covered in literature:

1. Atmospheric Turbulence Modeling for Aero Vehicles; Fractional Order Fits, NASA TM – 2010-216961
2. Modeling of Atmospheric Turbulence as Disturbances for Control Design and Evaluation of High Speed Propulsion Systems. *Journal of Dynamic Systems, Measurement, and Control* **134**:2, 021009, Jan-2012.

Fractional Order Modeling Approach

Example TF fits based on the derived model of first order pole-zero pairs, with 4 poles and 3 zeroes (1 pole followed by 3 pole – zero pairs):

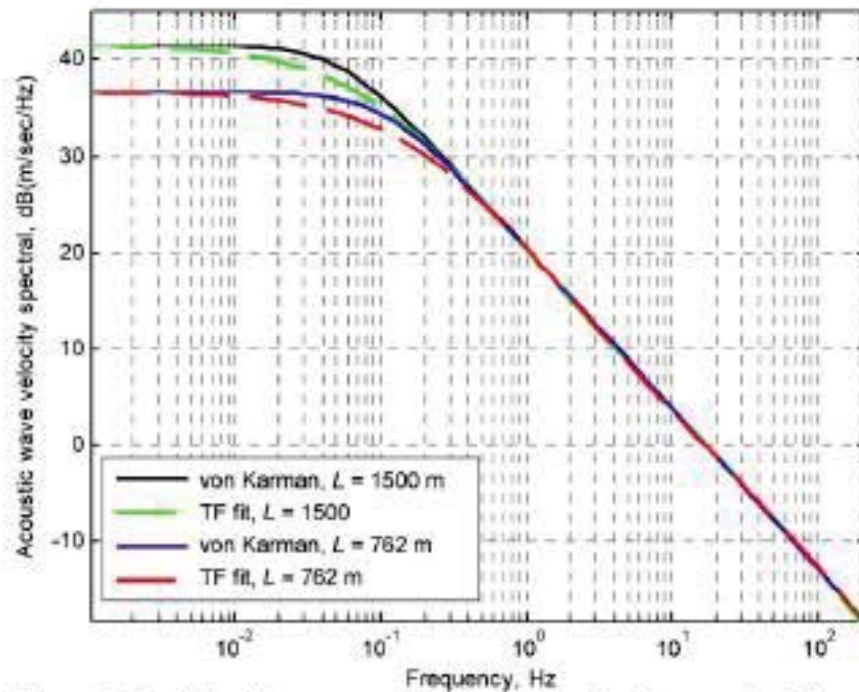


Figure C.9.—Von Karman acoustic wave velocity spectral due to temperature gust and its TF fits for different integral scale lengths ($\varepsilon = 8.6\text{e-}5 \text{ m}^2/\text{sec}^3$).

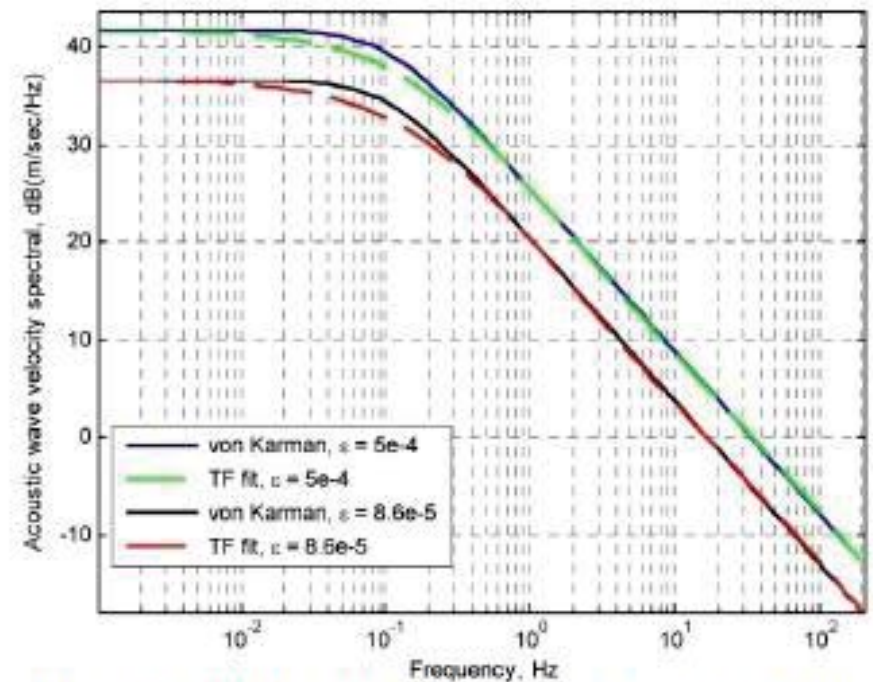


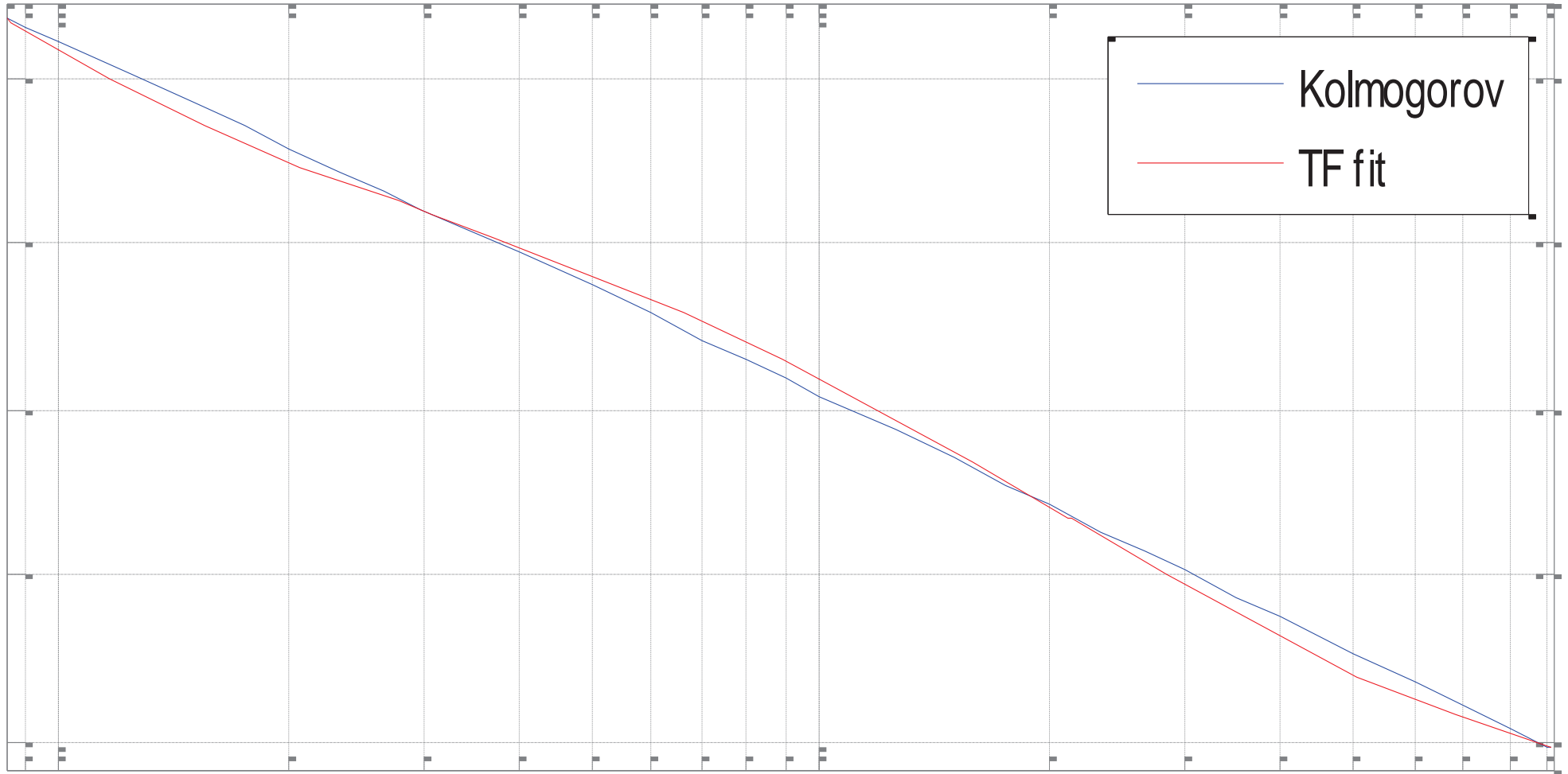
Figure C.10.—Von Karman acoustic wave velocity spectral due to temperature gust and its TF fits for different eddy dissipation rates ($L=762$ m).

- Note that differences in the integral scale lengths only impacts the low frequency range, where typical control system designs should have no problem to attenuate disturbances in this range.



Fractional Order Modeling Approach

With zoom –in: The staircase like approximation can be seen





Fractional Order Modeling Approach

- For control systems design this fractional order **filter model** is simplified by eliminating the integral scale length (using $L=762$ as the basis – considered standard in airplane industry according to Tank)

Wind gust longitudinal

$$G_{LA}(s) = 70\varepsilon^{2/9} \frac{(s/9.2 + 1)(s/55.0 + 1)(s/335.5 + 1)}{(s/1.46 + 1)(s/30.1 + 1)(s/85.7 + 1)(s/1593.1 + 1)}$$

Temperature

$$G_T(s) = 943\varepsilon^{2/6} \frac{(s/33.0 + 1)(s/45.6 + 1)(s/602.4 + 1)}{(s/1.1 + 1)(s/25.1 + 1)(s/109.8 + 1)(s/816.3 + 1)}$$

Longitudinal gusts due to temperature

$$G_{TLA}(s) = 472\varepsilon^{2/6} \frac{(M-1)}{\sqrt{M^2-1}} \frac{M\gamma R}{a_o} \frac{(s/33.0 + 1)(s/45.6 + 1)(s/602.4 + 1)}{(s/1.1 + 1)(s/25.1 + 1)(s/109.8 + 1)(s/816.3 + 1)}$$

- Similar for transverse wind gusts and pressure TFs
- Modeling of Atmospheric Turbulence as Disturbances for Control Design and Evaluation of High Speed Propulsion Systems. *Journal of Dynamic Systems, Measurement, and Control* **134**:2, 021009, Jan-2012.



Considerations for Atmospheric Turbulence Specifications

- Good Source of information in this area for a statistical approach is the publication “Atmospheric Disturbance Environmental Definition,” by Tank W. G., *NASA CR-195315, 1994*.
- This area also covered (worst case – non statistical) in article “Modeling of Atmospheric Turbulence as Disturbance for Control Design and Evaluation of High Speed Propulsion Systems” *ASME Journal of Dynamic Systems, measurement and control, 2011*.
 - Assumes that if a vehicle encounters an atmospheric turbulence, a critical Reynolds number will be exceeded and an eddy dissipation rate associated with the severity of the turbulence and corresponding wind gusts will be experienced.
 - This seemingly relaxes the necessity for statistical exceedance rates, which are based on pure chance and doesn't take into account the weather conditions a vehicle is flying in at any given moment in time.



Considerations for Atmospheric Turbulence Specifications

Continued from previous pg.

- Assumes worst case turbulence is a combination of wind gusts, temperature and pressure disturbances. Even tough, this is left to the user to define what is worst case.
- Spectrum frequency range ($f_{range} = \frac{V_{vehicle}}{\lambda_s}$) depends on the smallest length scales of turbulence (25m), before viscous dissipation starts to become significant.
- Spectrum sinusoid density chosen such maximum wind gust will not exceed a maximum (like 100 miles/hour – but user defined).
- Spectrum sinusoids equally spaced, or purposely coincide with known vehicle modes (guidelines provided, but user defined)



- Applied to the control design of a mixed compression inlet

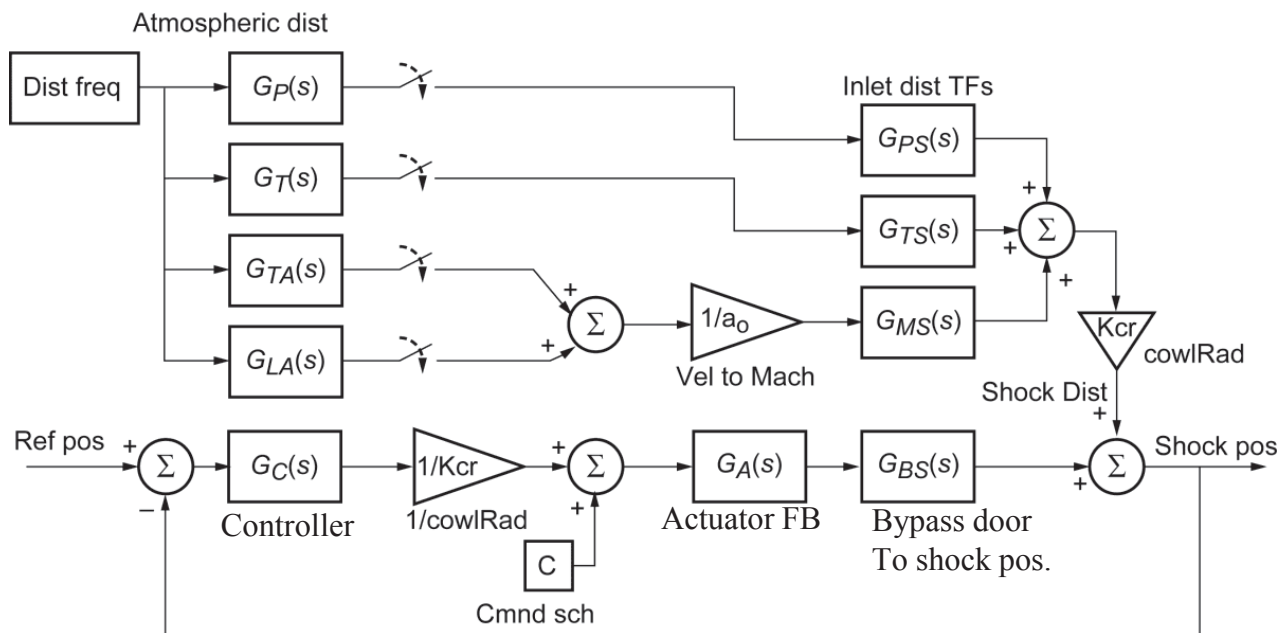
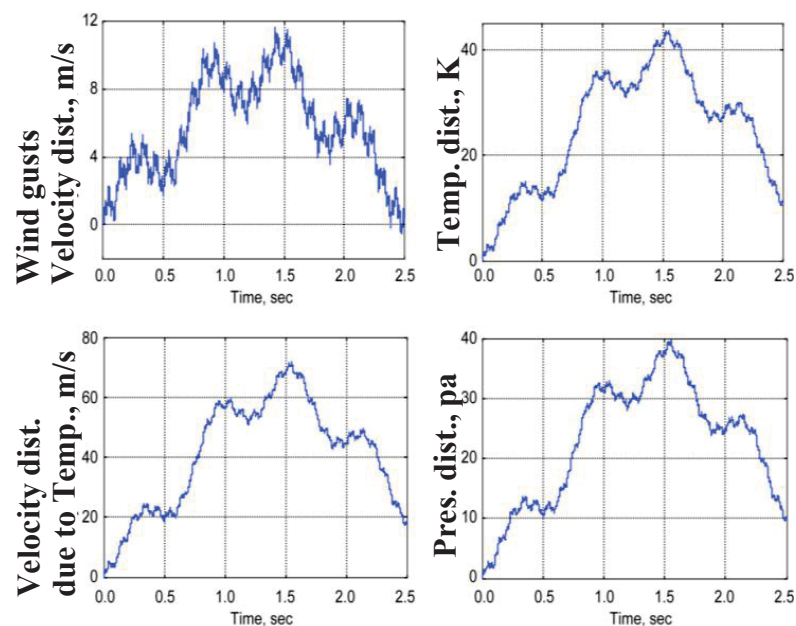
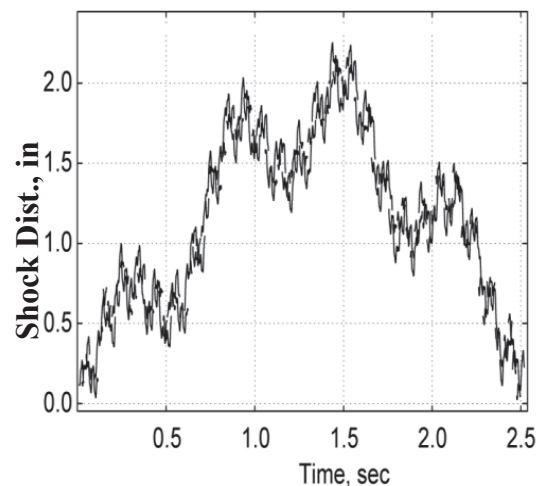


Figure 12.—Feedback control diagram of inlet shock position system via bypass door actuation.

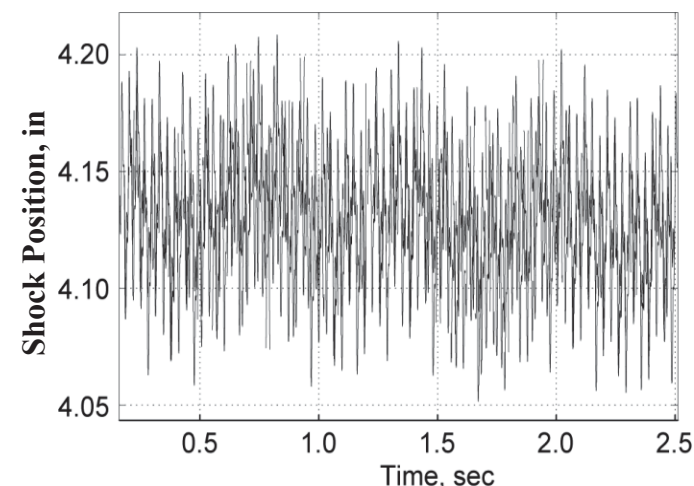


Atmospheric turbulence



Shock position disturbance

$$G_C(s) = \frac{100(s/2\pi 5 + 1)(s/2\pi 18.8 + 1)}{s(s/2\pi 14.5 + 1)(s/2\pi 104 + 1)} \cdot \frac{(s^2/(2\pi 145)^2 + 0.98s/2\pi 145 + 1)}{(s^2/(2\pi 5000)^2 + 1.4s/2\pi 5000 + 1)} \cdot \frac{(s^2/(2\pi 250)^2 + 1.3s/2\pi 250 + 1)}{(s^2/(2\pi 5500)^2 + 1.4s/2\pi 5500 + 1)} \cdot \frac{(s^2/(2\pi 350)^2 + 1.36s/2\pi 350 + 1)}{(s^2/(2\pi 6000)^2 + 1.4s/2\pi 6000 + 1)} \cdot \frac{(s^2/(2\pi 450)^2 + 1.0s/2\pi 450 + 1)}{(s^2/(2\pi 6500)^2 + 1.4s/2\pi 6500 + 1)} \cdot \frac{(s^2/(2\pi 550)^2 + 1.0s/2\pi 550 + 1)}{(s^2/(2\pi 7000)^2 + 1.4s/2\pi 7000 + 1)} \cdot \frac{(s^2/(2\pi 650)^2 + 0.60s/2\pi 650 + 1)}{(s^2/(2\pi 7500)^2 + 1.4s/2\pi 7500 + 1)}$$

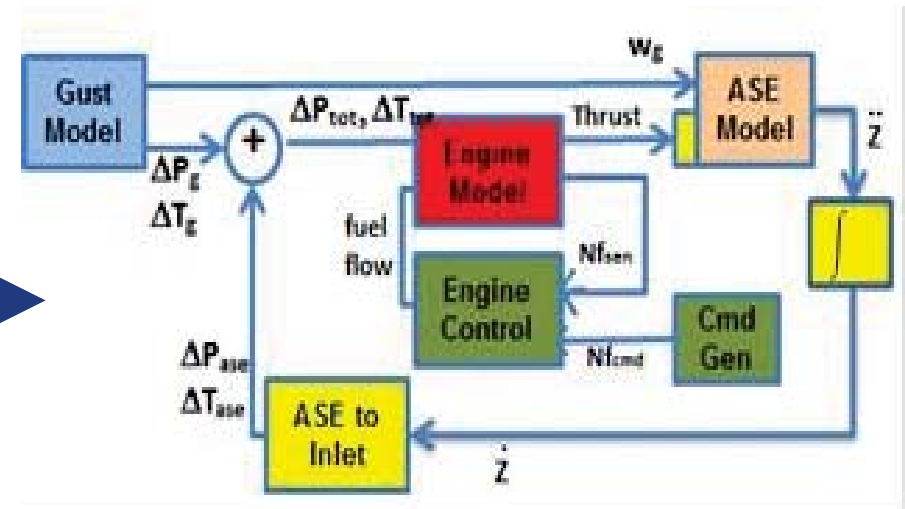
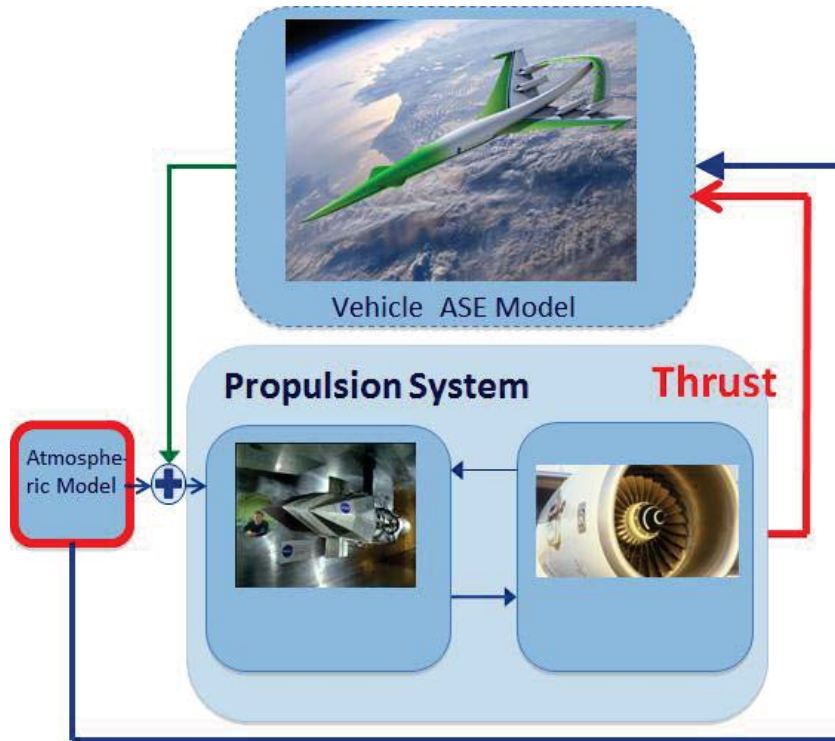


Controlled shock position

Applications

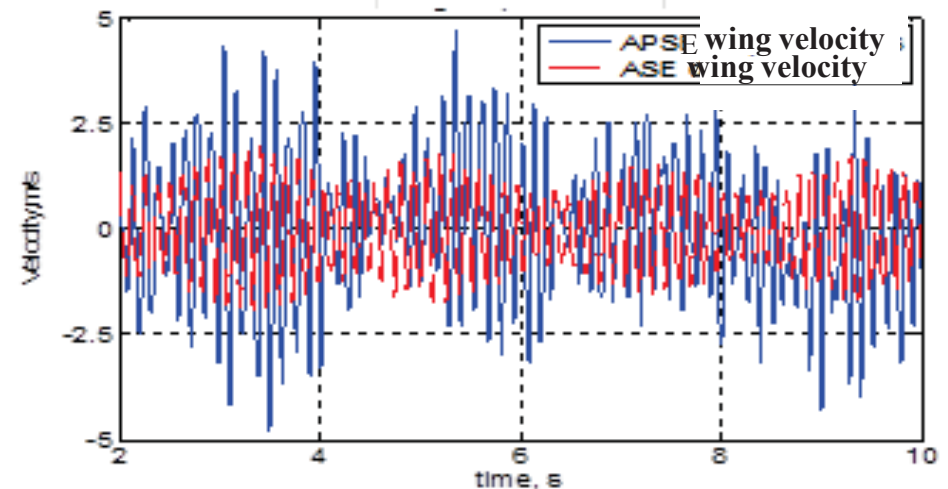
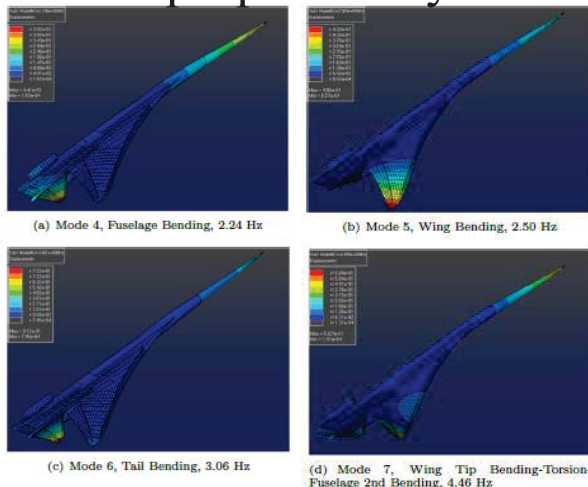


- Applied to AeroPropulsoServoElasticity (APSE) – closed loop coupling of propulsion and structural vehicle dynamics.



Closed-loop diagram of APSE simulation

Vehicle & propulsion system modeling

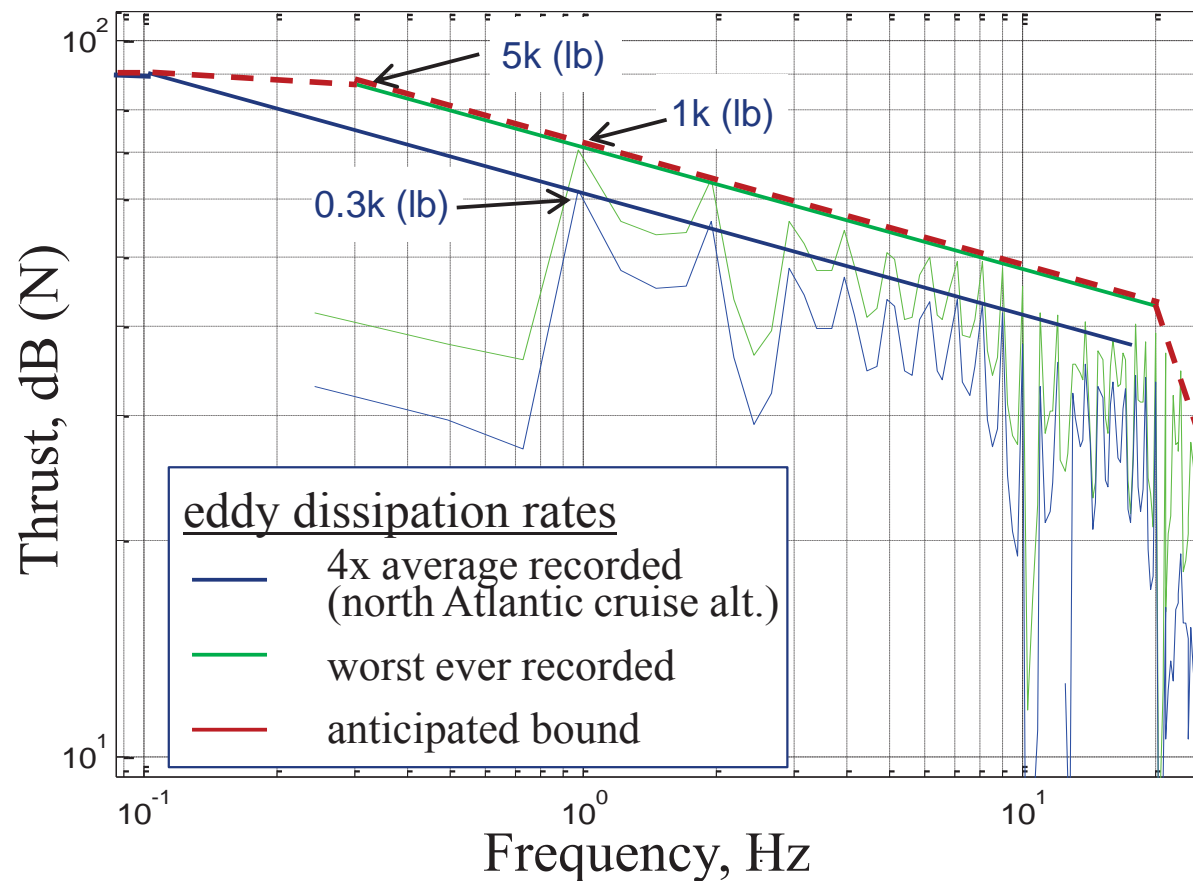


Wing velocity at vicinity of engine inlet for APSE system and for ASE alone

Applications



- Applied to assessments of vehicle thrust forces spectrum for different eddy dissipation rates – engine speed control, but no inlet shock control (external compression inlet)



Conclusions



- Covered a new modeling approach for atmospheric turbulence for aero vehicles.
 - Approach models turbulence in their natural fractional order form.
 - Approach is more accurate than traditional models like the Dryden model, which is widely used by industry and military - especially for high speed vehicles.
 - Probabilistic exceedance analysis can be conducted with this new approach as in the past. Even though, this model was developed for deterministic analysis in mind (i.e. for vehicle designed to handle specified worst case turbulence conditions).
 - Considerations for atmospheric turbulence specifications also provided in this approach. Even though, this is done independently of the methodology.
 - US patent has been filed.



Thank You!



Questions?